

# Novel Subcarrier-pair based Opportunistic DF Protocol for Cooperative Downlink OFDMA

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**Abstract**—A novel subcarrier-pair based opportunistic DF protocol is proposed for cooperative downlink OFDMA transmission aided by a decode-and-forward (DF) relay. Specifically, user message bits are transmitted in two consecutive equal-duration time slots. A subcarrier in the first slot can be paired with a subcarrier in the second slot for the DF relay-aided transmission to a user. In particular, the source and the relay can transmit simultaneously to implement beamforming at the subcarrier in the second slot for the relay-aided transmission. Each unpaired subcarrier in either the first or second slot is used by the source for direct transmission to a user without the relay's assistance. The sum rate maximized resource allocation (RA) problem is addressed for this protocol under a total power constraint. It is shown that the novel protocol leads to a maximum sum rate greater than or equal to that for a benchmark one, which does not allow the source to implement beamforming at the subcarrier in the second slot for the relay-aided transmission. Then, a polynomial-complexity RA algorithm is developed to find an (at least approximately) optimum resource allocation (i.e., source/relay power, subcarrier pairing and assignment to users) for either the proposed or benchmark protocol. Numerical experiments illustrate that the novel protocol can lead to a much greater sum rate than the benchmark one.<sup>1</sup>

## I. INTRODUCTION

The incorporation of subcarrier-pair based decode-and-forward (DF) relaying into orthogonal frequency division modulation (OFDM) or multiple-access (OFDMA) transmission and associated resource allocation (RA) were studied in [1]–[4] when the source-to-destination (S-D) link exists. In [1]–[3], an “always-relaying” DF protocol was used, i.e., a subcarrier in the first time slot is always paired with a subcarrier in the second slot for the relay-aided transmission. To better exploit the frequency-selective fading, we have proposed an opportunistic DF relaying protocol (sometimes termed as selection relaying) in [4]–[6], i.e., a subcarrier in the first time slot can either be paired with a subcarrier in the second slot for the relay-aided transmission, or used for the S-D direct transmission without the relay's assistance. It is very important to note that when some subcarriers in the first slot are used for the direct transmission, some subcarriers in the second slot will not be used, which wastes spectrum resource.

To address the above issue, we have proposed an improved DF protocol in [7]. This protocol is the same as those

considered in [4] except that the source can also make direct S-D transmission at every unpaired subcarrier in the second slot. This protocol and its RA were later intensively studied, e.g., in [7]–[12]. Note that *the improved protocol does not really improve the way how DF relaying is implemented over a subcarrier pair, but rather let the source utilize the unpaired subcarriers in the second slot for direct transmission to avoid the waste of spectrum resource*. In [11]–[13], the subcarrier pairing and power allocation are jointly optimized for point-to-point OFDM transmission. As for OFDMA systems, RA problems considering the joint optimization of power allocation and subcarrier assignment to users are addressed in [7]–[10]. In these works, a priori and CSI-independent subcarrier pairing is considered, i.e., a subcarrier in the first slot is always paired with the same subcarrier in the second slot if the relay-aided mode is used. It is a complicated RA problem to jointly optimize subcarrier pairing, power allocation and subcarrier assignment to users.

In this paper, we consider downlink OFDMA transmission from a source to multiple users aided by a DF relay. Compared with the existing works, this paper makes the following contributions. First, a novel subcarrier-pair based opportunistic DF relaying protocol is proposed. A benchmark protocol using the improved protocol as in [7] is also considered. However, the proposed protocol uses further improved relay-aided transmission, which allows the source and relay to transmit simultaneously to implement beamforming at the subcarrier in the second slot. *Note that the proposed protocol truly improves the implementation of DF relaying over a subcarrier pair with transmit beamforming, which is not the case for the benchmark protocol*. Second, the sum rate maximized RA problem is addressed for both the novel and benchmark protocols, under a total power constraint for the whole system. It is shown that the novel protocol leads to a maximum sum rate greater than or equal to that for the benchmark one. An RA algorithm is developed for each protocol to find the globally optimum source/relay power allocation and subcarrier pairing to maximize the sum rate of all users.

The rest of this paper is organized as follows. In the next section, the system and transmission protocols are described. In Section III-A, we will focus on computing the maximum rate and optimum power allocation for a subcarrier pair using the relay-aided transmission for both protocols. Using these results, an RA algorithm will be developed in Section III, and

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numerical experiments are shown in Section IV. Finally, some conclusions are drawn.

Notations: A letter in bold, e.g.  $\mathbf{x}$ , represents a set.  $\mathcal{C}(x) = \frac{1}{2} \log_2(1+x)$  and  $[x]^+ = \max\{x, 0\}$ .

## II. PROTOCOLS AND WSR MAXIMIZATION PROBLEM

### A. The transmission system and protocols

Consider the downlink OFDMA transmission from a source to  $U$  users collected in the set  $\mathbf{U} = \{u|u = 1, \dots, U\}$  aided by a DF relay. The source and the relay can simultaneously emit OFDM symbols using  $K$  subcarriers and with sufficiently long cyclic prefix to eliminate inter-symbol interference. User message bits are transmitted in two consecutive equal-duration time slots, during which all channels are assumed to keep unchanged. During the first slot, only the source broadcasts  $N$  OFDM symbols. Both the relay and all users receive these symbols. After proper processing explained later, the source and relay simultaneously broadcast  $N$  OFDM symbols, and the users receive them during the second slot. Due to the OFDMA, each subcarrier is dedicated to transmitting a single user's message exclusively. A subcarrier in the first slot can be paired with a subcarrier in the second slot for the relay-aided mode transmission to a user. Each unpaired subcarrier in either the first or second slot is used by the source for the direct mode transmission to a user.

To simplify description, we use subcarriers  $k$  and  $l$  to denote the  $k$ th and  $l$ th subcarriers used during the first and second slots, respectively ( $k, l = 1, \dots, K$ ). We define the source transmission powers for subcarrier  $k$  in the first slot and subcarrier  $l$  in the second slot as  $P_{s,k,1}$  and  $P_{s,l,2}$ , respectively. The relay transmission power for subcarrier  $l$  is  $P_{r,l,2}$ . The complex amplitude gains at subcarrier  $k$  for the source-to-relay, source-to- $u$  and relay-to- $u$  channels are  $h_{sr,k}$ ,  $h_{su,k}$  and  $h_{ru,k}$ , respectively. The two transmission modes for the novel protocol are elaborated as follows:

1) *The relay-aided transmission mode:* Suppose subcarrier  $k$  is paired with subcarrier  $l$  for the relay-aided mode transmission to user  $u$ . A block of message bits are first encoded into a code word of complex symbols  $\{\theta(n)|n = 1, \dots, N\}$  with  $E(|\theta(n)|^2) = 1, \forall n$ . In the first slot, the source broadcasts the codeword over subcarrier  $k$  as illustrated in Figure 1.a. At the relay and user  $u$ , the  $n$ th baseband signals received through subcarrier  $k$  are

$$y_{r,k}(n) = \sqrt{P_{s,k,1}} h_{sr,k} \theta(n) + z_{r,k}(n), n = 1, \dots, N, \quad (1)$$

and

$$y_{u,k,1}(n) = \sqrt{P_{s,k,1}} h_{su,k} \theta(n) + z_{u,k,1}(n), n = 1, \dots, N, \quad (2)$$

respectively, where  $z_{r,k}(n)$  and  $z_{u,k,1}(n)$  are both additive white Gaussian noise (AWGN) with power  $\sigma^2$ . The signal-to-noise ratio (SNR) at the relay is  $P_{s,k,1} G_{sr,k}$  where  $G_{sr,k} = \frac{|h_{sr,k}|^2}{\sigma^2}$ . At the end of the first time slot, the relay decodes the message bits from  $\{y_{r,k}(n)|n = 1, \dots, N\}$  and then

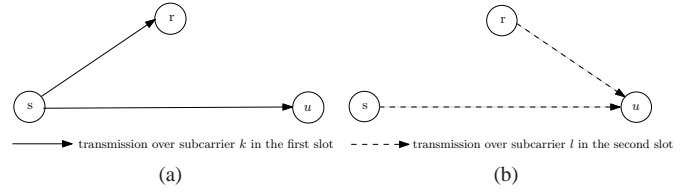


Fig. 1. The relay-aided transmission mode over the subcarrier pair  $(k, l)$  to user  $u$ .

reencodes those bits into the same codeword as the source did.

In the second time slot, the source and relay broadcast the codewords  $\{\theta(n)e^{-j\angle h_{su,l}}|\forall n\}$  and  $\{\theta(n)e^{-j\angle h_{ru,l}}|\forall n\}$  through subcarrier  $l$ , respectively, where  $\angle h_{su,l}$  and  $\angle h_{ru,l}$  represent the phase of  $h_{su,l}$  and  $h_{ru,l}$ , respectively. This means that the source and relay implement transmit beamforming to emit the codeword through subcarrier  $l$  as illustrated in Figure 1.b. Note that the source and relay need to know the phase of  $h_{su,l}$  and  $h_{ru,l}$ , respectively. At user  $u$ , the  $n$ th baseband signal received through subcarrier  $l$  is

$$y_{u,l,2}(n) = (\sqrt{P_{s,l,2}}|h_{su,l}| + \sqrt{P_{r,l,2}}|h_{ru,l}|)\theta(n) + z_{u,l,2}(n), \quad (3)$$

where  $z_{u,l,2}(n)$  is the AWGN with power  $\sigma^2$ .

Finally, user  $u$  decodes the message bits from all signals received during the two slots. These signals can be grouped into  $N$  vectors, the  $n$ th of which is

$$\begin{aligned} \mathbf{y}(n) &= \begin{bmatrix} y_{u,k,1}(n) \\ y_{u,l,2}(n) \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{P_{s,k,1}} h_{su,k} \\ \sqrt{P_{s,l,2}}|h_{su,l}| + \sqrt{P_{r,l,2}}|h_{ru,l}| \end{bmatrix} \theta(n) + \mathbf{z}(n), \end{aligned} \quad (4)$$

where  $\mathbf{z}(n) = [z_{u,k,1}(n), z_{u,l,2}(n)]^T$ . Note that the transmission of the codeword in effect makes  $N$  uses of a discrete memoryless single-input-two-output channel specified by (4), with the  $n$ th input and output being  $\theta(n)$  and  $\mathbf{y}(n)$ , respectively. To achieve the maximum reliable transmission rate, maximum ratio combining should be used [14]. It can readily be derived that the SNR for after this combining is

$$\gamma_{klu}(P_{s,k,1}, P_{s,l,2}, P_{r,l,2}) = G_{su,k} P_{s,k,1} + (\sqrt{G_{su,l} P_{s,l,2}} + \sqrt{G_{ru,l} P_{r,l,2}})^2, \quad (5)$$

where  $G_{su,k} = \frac{|h_{su,k}|^2}{\sigma^2}$  and  $G_{ru,l} = \frac{|h_{ru,l}|^2}{\sigma^2}$ . To ensure both the relay and user  $u$  can reliably decode the message bits, the maximum number of message bits that can be transmitted is  $2NC(G_{sr,k} P_{s,k,1})$  and  $2NC(\gamma_{klu}(P_{s,k,1}, P_{s,l,2}, P_{r,l,2}))$ , respectively. This means that the maximum transmission rate over the subcarrier pair  $(k, l)$  in the relay-aided mode to user  $u$  is equal to  $\mathcal{C}(\min\{G_{sr,k} P_{s,k,1}, \gamma_{klu}(P_{s,k,1}, P_{s,l,2}, P_{r,l,2})\})$  bits/OFDM-symbol (bps).

2) *The direct transmission mode:* Suppose subcarrier  $k$  (respectively, subcarrier  $l$ ) is unpaired with any subcarrier in the second (respectively, first) slot, and is used for direct mode transmission to user  $u$ . The source first encodes message bits

into a codeword of  $N$  symbols, which are then broadcast through subcarrier  $k$  (respectively, subcarrier  $l$ ). In such a case, the relay keeps silent at subcarrier  $l$  in the second slot, i.e.  $P_{r,l,2} = 0$ ). User  $u$  decodes the message bits from the signals received through subcarrier  $k$  (respectively, subcarrier  $l$ ). The maximum rate through subcarrier  $k$  (respectively, subcarrier  $l$ ) in the direct transmission mode is  $\mathcal{C}(P_{s,k,1}G_{su,k})$  (respectively,  $\mathcal{C}(P_{s,l,2}G_{su,l})$ ) bpos.

A benchmark protocol is also considered. This protocol is the same as the novel protocol except for the relay-aided transmission mode. Specifically, the relay-aided mode is the same as that widely studied in the literature, i.e., the source does not transmit at subcarrier  $l$  during the second slot, if subcarriers  $k$  and  $l$  are paired for the relay-aided transmission to user  $u$ . In such a case, the maximum rate for the relay-aided transmission over that subcarrier pair to user  $u$  is equal to  $\mathcal{C}(\min\{G_{sr,k}P_{s,k,1}, G_{su,k}P_{s,k,1} + G_{ru,l}P_{r,l,2}\})$  bpos.

### B. The sum rate maximization problem

We assume there exists a central controller which knows precisely the CSI  $\{G_{sr,k}, G_{su,k}, G_{ru,k} | \forall k\}$ . Before the data transmission, the controller needs to find the optimum subcarrier assignment and power allocation to maximize the sum rate of all users for the adopted transmission protocol (which can be either the novel or benchmark protocol), when the total power consumption is not higher than a prescribed value  $P_t$ . Then, the controller can inform the source and the relay the optimum RA to be adopted for data transmission.

It can be shown that the novel protocol leads to a maximum sum rate greater than or equal to that for the benchmark one. This can be proven as follows. Note that the benchmark protocol is a special case of the novel one, since it is equivalent to the novel one constrained with  $P_{s,l,2} = 0$  if subcarrier  $l$  is paired with a subcarrier for the relay-aided transmission. Suppose the optimum subcarrier assignment and power allocation have been found for the benchmark protocol. By using the novel protocol with the same subcarrier assignment and power allocation, the same sum rate can be achieved. Obviously, the maximum sum rate for the novel protocol is greater than or equal to that sum rate, namely the maximum sum rate for the benchmark protocol.

## III. RA ALGORITHM DESIGN

### A. Rate maximization for the relay-aided mode over a subcarrier pair

Assume subcarriers  $k$  and  $l$  are paired for the relay-aided mode transmission to user  $u$ , and a sum power  $P$  is used for this pair. For the benchmark protocol, it can be shown by using an intuitive method similar as the one introduced in Appendix of [9] that the maximum rate associated with the above optimum solution is equal to  $\mathcal{C}(G_{klu}^b P)$  with

$$G_{klu}^b = \begin{cases} \frac{G_{sr,k}G_{ru,l}}{\Delta_{u,k} + G_{ru,l}} & \text{if } \min\{G_{sr,k}, G_{ru,l}\} > G_{su,k}, \\ \min\{G_{sr,k}, G_{su,k}\} & \text{if } \min\{G_{sr,k}, G_{ru,l}\} \leq G_{su,k}, \end{cases} \quad (6)$$

and the optimum  $P_{s,k,1}$  is

$$P_{s,k,1} = \begin{cases} \frac{G_{ru,l}}{\Delta_{u,k} + G_{ru,l}} P & \text{if } \min\{G_{sr,k}, G_{ru,l}\} > G_{su,k}, \\ P & \text{if } \min\{G_{sr,k}, G_{ru,l}\} \leq G_{su,k}. \end{cases}$$

To facilitate the derivation for the proposed protocol, define  $\Delta_{u,k} = G_{sr,k} - G_{su,k}$  and  $G_{u,l} = G_{su,l} + G_{ru,l}$ . To maximize the rate, the optimum  $P_{s,k,1}$ ,  $P_{s,l,2}$  and  $P_{r,l,2}$  are the optimum solution for

$$\begin{aligned} \max_{P_{s,k,1}, P_{s,l,2}, P_{r,l,2}} \quad & \min\{G_{sr,k}P_{s,k,1}, \gamma_{klu}(P_{s,k,1}, P_{s,l,2}, P_{r,l,2})\} \\ \text{s.t.} \quad & P_{s,k,1} + P_{s,l,2} + P_{r,l,2} = P, \\ & P_{s,k,1} \geq 0, P_{s,l,2} \geq 0, P_{r,l,2} \geq 0. \end{aligned} \quad (7)$$

Using Cauchy-Schwartz inequality and an intuitive method similar as that introduced in Appendix of [9], it can be shown that the optimum  $P_{s,k,1}$ ,  $P_{s,l,2}$  and  $P_{r,l,2}$  for (7) are

$$\begin{aligned} P_{s,k,1} &= \begin{cases} \frac{G_{u,l}}{\Delta_{u,k} + G_{u,l}} P & \text{if } \min\{G_{sr,k}, G_{u,l}\} > G_{su,k}, \\ P & \text{if } \min\{G_{sr,k}, G_{u,l}\} \leq G_{su,k}, \end{cases} \\ P_{s,l,2} &= \begin{cases} \frac{G_{su,l}}{G_{u,l}} \frac{\Delta_{u,k}}{(\Delta_{u,k} + G_{u,l})} P & \text{if } \min\{G_{sr,k}, G_{u,l}\} > G_{su,k}, \\ 0 & \text{if } \min\{G_{sr,k}, G_{u,l}\} \leq G_{su,k}, \end{cases} \end{aligned}$$

and  $P_{r,l,2} = P - P_{s,k,1} - P_{s,l,2}$  (please see [15] for more details). The maximum rate associated with the above optimum solution is equal to  $\mathcal{C}(G_{klu}^n P)$  with

$$G_{klu}^n = \begin{cases} \frac{G_{sr,k}G_{u,l}}{\Delta_{u,k} + G_{u,l}} & \text{if } \min\{G_{sr,k}, G_{u,l}\} > G_{su,k}, \\ \min\{G_{sr,k}, G_{su,k}\} & \text{if } \min\{G_{sr,k}, G_{u,l}\} \leq G_{su,k}. \end{cases} \quad (8)$$

### B. Formulation of the WSR maximization problem

For the adopted protocol (which can be either the proposed or benchmark protocol), we first define

$$G_{klu} = \begin{cases} G_{klu}^n & \text{if the proposed protocol is adopted,} \\ G_{klu}^b & \text{if the benchmark protocol is adopted.} \end{cases}$$

For any possible subcarrier assignment used by the adopted protocol, suppose  $m$  subcarrier pairs are assigned to the relay-aided transmission, the unpaired subcarriers in the two slots can always be one-to-one associated with each other to form  $K - m$  virtual subcarrier pairs for the direct transmission. Based on this observation, we define:

- $t_{klu} \in \{0, 1\}$  and  $P_{klu} \geq 0$ ,  $\forall k, l, u$ .  $t_{klu} = 1$  indicates that subcarrier  $k$  is paired with subcarrier  $l$  for the relay-aided transmission to user  $u$ . When  $t_{klu} = 1$ ,  $P_{klu}$  is used as the total power for the subcarrier pair  $(k, l)$ .
- $t_{klab} \in \{0, 1\}$ ,  $\alpha_{klab} \geq 0$  and  $\beta_{klab} \geq 0$ ,  $\forall k, l, u$ .  $t_{klab} = 1$  indicates that subcarrier  $k$  is assigned in the direct transmission mode to user  $a$  during the first slot, and so is subcarrier  $l$  to user  $b$  during the second slot. When  $t_{klab} = 1$ ,  $P_{s,k,1}$  and  $P_{s,l,2}$  take the value of  $\alpha_{klab}$  and  $\beta_{klab}$ , respectively.

Let us collect all indicator and power variables in the sets  $\mathbf{I}$  and  $\mathbf{P}$ , respectively, and define  $\mathbf{S} = \{\mathbf{I}, \mathbf{P}\}$ . Every feasible RA scheme can be described by an  $\mathbf{S}$  satisfying simultaneously

$$t_{klu}, t_{klab} \in \{0, 1\}, \forall k, l, u, a, b, \quad (9)$$

$$\sum_l \left( \sum_u t_{klu} + \sum_{a,b} t_{klab} \right) = 1, \forall k, \quad (10)$$

$$\sum_k \left( \sum_u t_{klu} + \sum_{a,b} t_{klab} \right) = 1, \forall l, \quad (11)$$

$$\sum_{k,l,u,a,b} (t_{klu} P_{klu} + t_{klab} (\alpha_{klab} + \beta_{klab})) \leq P_t, \quad (12)$$

$$P_{klu} \geq 0, \alpha_{klab} \geq 0, \beta_{klab} \geq 0, \forall k, l, u, a, b. \quad (13)$$

Given a feasible  $\mathbf{S}$ , the maximum sum rate for the adopted protocol is

$$f(\mathbf{S}) = \sum_{k,l,u,a,b} (t_{klu} \mathcal{C}(G_{klu} P_{klu}) + t_{klab} (\mathcal{C}(G_{sa,k} \alpha_{klab}) + \mathcal{C}(G_{sb,l} \beta_{klab}))), \quad (14)$$

and the sum rate maximization problem is to solve

$$\begin{aligned} \max_{\mathbf{S}} \quad & f(\mathbf{S}) \\ \text{s.t.} \quad & (9) - (13) \end{aligned} \quad (15)$$

for a globally optimum  $\mathbf{S}$ . Obviously, (15) is a nonconvex mixed-integer nonlinear program. To find a globally optimum  $\mathbf{S}$ , all indicator variables are first relaxed to be continuous within  $[0, 1]$ , i.e., Then, we make the COV from  $\mathbf{P}$  to  $\tilde{\mathbf{P}} = \{\tilde{P}_{klu}, \tilde{\alpha}_{klab}, \tilde{\beta}_{klab} | \forall k, l, u, a, b\}$ , where every  $\tilde{P}_{klu}$ ,  $\tilde{\alpha}_{klab}$  and  $\tilde{\beta}_{klab}$  satisfy, respectively,

$$\tilde{P}_{klu} = t_{klu} P_{klu}, \tilde{\alpha}_{klab} = t_{klab} \alpha_{klab}, \tilde{\beta}_{klab} = t_{klab} \beta_{klab}.$$

After collecting all variables into  $\mathbf{X} = \{\mathbf{I}, \tilde{\mathbf{P}}\}$ , the RA problem can be rewritten as

$$\max_{\mathbf{X}} \quad g(\mathbf{X}) \quad (16)$$

$$\text{s.t.} \quad t_{klu}, t_{klab} \in [0, 1], \forall k, l, u, a, b. \quad (17)$$

$$\sum_{k,l,u,a,b} (\tilde{P}_{klu} + \tilde{\alpha}_{klab} + \tilde{\beta}_{klab}) \leq P_t, \quad (18)$$

$$\tilde{P}_{klu} \geq 0, \tilde{\alpha}_{klab} \geq 0, \tilde{\beta}_{klab} \geq 0, \forall k, l, u, a, b, \quad (19)$$

where  $g(\mathbf{X})$  represents the maximum sum rate expressed as

$$g(\mathbf{X}) = \sum_{k,l,u,a,b} (\phi(t_{klu}, \tilde{P}_{klu}, G_{klu}) + \phi(t_{klab}, \tilde{\alpha}_{klab}, G_{sa,k}) + \phi(t_{klab}, \tilde{\beta}_{klab}, G_{sb,l})), \quad (20)$$

and

$$\phi(t, x, C) = \begin{cases} t \cdot \mathcal{C}(C \frac{x}{t}) & \text{if } t > 0, \\ 0 & \text{if } t = 0. \end{cases} \quad (21)$$

Obviously, (16) is a relaxation of (15). We will find an (at least approximately) optimum solution for (16), and show that the  $\mathbf{S}$  corresponding to this solution is still feasible, and

hence (at least approximately) optimum for (15), which can be shown in a similar way as reported in [15]. To this end, note that  $\phi(t, p, G)$  is a continuous and concave function if  $t \geq 0$  and  $x$ , because it is a perspective function of  $\mathcal{C}(Gp)$  which is concave of  $p$  [16]. As a result,  $g(\mathbf{X})$  is a concave function of  $\mathbf{X}$  in its feasible domain for (16). This means that (16) is a convex optimization problem. Apparently, it also satisfies the Slater constraint qualification, therefore its duality gap is zero, which justifies the use of the dual method to look for the globally optimum of (16), denoted by  $\mathbf{X}^*$  hereafter.

To use the dual method,  $\mu$  is introduced as a Lagrange multiplier for the constraint (18). The Lagrange relaxation problem (LRP) for (16) is

$$\begin{aligned} \max_{\mathbf{X}} \quad & L(\mu, \mathbf{X}) = g(\mathbf{X}) + \mu (P_t - \bar{P}(\mathbf{X})) \\ \text{s.t.} \quad & (17), (10) - (11), (19), \end{aligned} \quad (22)$$

where  $L(\mu, \mathbf{X})$  is the Lagrangian of (16) and  $\bar{P}(\mathbf{X})$  is the sum of all  $\tilde{P}_{klu}$ ,  $\tilde{\alpha}_{klab}$  and  $\tilde{\beta}_{klab}$  in  $\mathbf{X}$ . A global optimum of (22) is denoted by  $\mathbf{X}_\mu$ . The dual function is defined as  $d(\mu) = L(\mu, \mathbf{X}_\mu)$ . In particular,  $P_t - \bar{P}(\mathbf{X}_\mu)$  is a subgradient of  $d(\mu)$ , i.e., it satisfies  $\forall \mu', d(\mu') \geq d(\mu) + (\mu' - \mu)(P_t - \bar{P}(\mathbf{X}_\mu))$ , and the dual problem is to find the dual optimum  $\mu^* = \arg \min_{\mu \geq 0} d(\mu)$ .

Since (16) has zero duality gap, it satisfies two important properties. First,  $\mu^* > 0$ . This is because  $\mu^*$  represents the sensitivity of the optimum objective value for (16) with respect to  $P_t$ , i.e.,  $\frac{g(\mathbf{X}^*)}{P_t} = \mu^*$  [16]. Obviously,  $g(\mathbf{X}^*)$  is strictly increasing of  $\tilde{P}_t$ , meaning that  $\mu^* > 0$ . Second,  $\mu$  and  $\mathbf{X}$  are equal to  $\mu = \mu^*$  and  $\mathbf{X}_\mu = \mathbf{X}^*$ , if and only if  $\mathbf{X}_\mu$  is feasible and  $\mu(P_t - \bar{P}(\mathbf{X}_\mu)) = 0$  is satisfied according to Proposition 5.1.5 in [17]. Based on the above properties, the  $\mu > 0$  and  $\mathbf{X}_\mu$  satisfying  $\bar{P}(\mathbf{X}_\mu) = P_t$  can be found as  $\mu^*$  and  $\mathbf{X}^*$ . Therefore, the key to developing a duality based algorithm consists of two procedures to find  $\mu^*$  and  $\mathbf{X}_\mu$ , respectively. We first introduce the one to find  $\mathbf{X}_\mu$  as follows.

1) *Finding  $\mathbf{X}_\mu$  when  $\mu > 0$* : The following strategy is used to find  $\mathbf{X}_\mu$  for (22) when  $\mu > 0$ . First, the optimum  $\tilde{\mathbf{P}}$  for (22) with fixed  $\mathbf{I}$  is found and denoted by  $\tilde{\mathbf{P}}_{\mathbf{I}}$ . Define  $\mathbf{X}_{\mathbf{I}} = \{\mathbf{I}, \tilde{\mathbf{P}}_{\mathbf{I}}\}$ . Then we find the optimum  $\mathbf{I}$  to maximizing  $L(\mu, \mathbf{X}_{\mathbf{I}})$  subject to (17), (10) and (11). Finally,  $\mathbf{X}_{\mathbf{I}}$  corresponding to this optimum  $\mathbf{I}$  can be taken as  $\mathbf{X}_\mu$ .

Suppose  $\mathbf{I}$  is fixed, we find  $\tilde{\mathbf{P}}_{\mathbf{I}}$  as follows. Specifically, every  $\tilde{P}_{klu}$  in  $\tilde{\mathbf{P}}_{\mathbf{I}}$  is equal to 0 when  $t_{klu} = 0$ . When  $t_{klu} > 0$ , the optimum  $\tilde{P}_{klu}$  can be found by using the KKT conditions related to  $\tilde{P}_{klu}$ . In summary, the optimum  $\tilde{P}_{klu}$  can be shown to be  $\tilde{P}_{klu} = t_{klu} \Lambda(\mu, G_{klu})$ , where  $\Lambda(\mu, G) = \left[ \frac{\log_2 e}{2\mu} - \frac{1}{G} \right]^+$ . In a similar way, the optimum  $\tilde{\alpha}_{klab}$  and  $\tilde{\beta}_{klab}$  can be shown to be  $\tilde{\alpha}_{klab} = t_{klab} \Lambda(\mu, G_{sa,k})$  and  $\tilde{\beta}_{klab} = t_{klab} \Lambda(\mu, G_{sb,l})$ , respectively. Using these formulas,  $\mathbf{X}_{\mathbf{I}} = \{\mathbf{I}, \tilde{\mathbf{P}}_{\mathbf{I}}\}$  can be found. It can readily be shown that

$$L(\mu, \mathbf{X}_{\mathbf{I}}) = \mu P_t + \sum_{k,l,u,a,b} (t_{klu} A_{klu} + t_{klab} B_{klab}) \quad (23)$$



where

$$A_{klu} = \mathcal{C}(G_{klu}\Lambda(\mu, G_{klu}) - \mu \cdot \Lambda(\mu, G_{klu}))$$

and

$$B_{klab} = \mathcal{C}(G_{sa,k}\Lambda(\mu, G_{sa,k}) - \mu \cdot \Lambda(\mu, G_{sa,k})) + \mathcal{C}(G_{sb,l}\Lambda(\mu, G_{sb,l}) - \mu \cdot \Lambda(\mu, G_{sb,l})).$$

Finally, we find the optimum  $\mathbf{I}$  for maximizing  $L(\mu, \mathbf{X}_\mathbf{I})$  subject to (17), (10) and (11). This problem is equivalent to solving

$$\begin{aligned} \max_{\mathbf{I}, \{t_{kl} | \forall k, l\}} \quad & \sum_{k,l} \sum_{u,a,b} (t_{klu} A_{klu} + t_{klab} B_{klab}) \\ \text{s.t.} \quad & \sum_l t_{kl} = 1, \forall k, \quad \sum_k t_{kl} = 1, \forall l, \\ & t_{kl} = \sum_u t_{klu} + \sum_{a,b} t_{klab}, \forall k, l. \\ & t_{klu} \geq 0, t_{klab} \geq 0, \forall k, l, u, a, b. \end{aligned} \quad (24)$$

Note that the inequality

$$\sum_{u,a,b} (t_{klu} A_{klu} + t_{klab} B_{klab}) \leq t_{kl} C_{kl} \quad (25)$$

holds where  $C_{kl} = \max\{\max_u A_{klu}, \max_{a,b} B_{klab}\}$ . Let us call  $A_{klu}$  as the metric for  $t_{klu}$  and  $B_{klab}$  as the metric for  $t_{klab}$ . This inequality is tightened when all entries of  $\{t_{klu}, t_{klab} | \forall u, a, b\}$  are assigned to zero, except that the one with the metric equal to  $C_{kl}$  is assigned to  $t_{kl}$ . Therefore, after the problem

$$\begin{aligned} \max_{\{t_{kl} | \forall k, l\}} \quad & \sum_{k,l} \sum_{u,a,b} t_{kl} C_{kl} \\ \text{s.t.} \quad & \sum_l t_{kl} = 1, \forall k, \quad \sum_k t_{kl} = 1, \forall l, \\ & t_{kl} \geq 0, \forall k, l, \end{aligned} \quad (26)$$

is solved for its optimum solution  $\{t_{kl}^* | \forall k, l\}$ , an optimum  $\mathbf{I}$  for (24) can be constructed by assigning for every combination of  $k$  and  $l$ , all entries in  $\{t_{klu}, t_{klab} | \forall u, a, b\} \subset \mathbf{I}$  to zero, except for the one with the metric equal to  $C_{kl}$  to  $t_{kl}^*$ .

Most interestingly, (26) is a standard assignment problem, hence  $\{t_{kl}^* | \forall k, l\}$  can be found efficiently by the Hungarian algorithm, and every entry in  $\{t_{kl}^* | \forall k, l\}$  is either 0 or 1 [18]. After knowing  $\{t_{kl}^* | \forall k, l\}$ , the optimum  $\mathbf{I}$  can be constructed according to the way mentioned earlier. Finally, the corresponding  $\mathbf{X}_\mathbf{I} = \{\mathbf{I}, \tilde{\mathbf{P}}_\mathbf{I}\}$  is assigned to  $\mathbf{X}_\mu$ . Note that the Hungarian algorithm to solve (26) has a complexity of  $O(K^3)$  [18], meaning that the complexity of finding  $\mathbf{X}_\mu$  is  $O(K^3)$ .

To find  $\mu^*$ , an iterative method which updates  $\mu$  with  $\mu = [\mu - \delta(P_t - \bar{P}(\mathbf{X}_\mu))]^+$  can be used, where  $\delta > 0$  is a prescribed step size [17]. However, this method converges very slowly, since  $\delta$  has to be very small to guarantee convergence. To develop a faster algorithm, we first prove that  $\bar{P}(\mathbf{X}_\mu)$  is a decreasing function of  $\mu > 0$ . To this end, suppose  $\mu_1 \geq \mu_2 > 0$ . Since  $P_t - \bar{P}(\mathbf{X}_{\mu_2})$  is a subgradient of  $d(\mu)$  at  $\mu$ , the

inequalities  $d(\mu_1) \geq d(\mu_2) + (\mu_1 - \mu_2)(P_t - \bar{P}(\mathbf{X}_{\mu_2}))$  and  $d(\mu_2) \geq d(\mu_1) + (\mu_2 - \mu_1)(P_t - \bar{P}(\mathbf{X}_{\mu_1}))$  follow. As a result,  $(\mu_1 - \mu_2)(P_t - \bar{P}(\mathbf{X}_{\mu_1})) \geq d(\mu_1) - d(\mu_2) \geq (\mu_1 - \mu_2)(P_t - \bar{P}(\mathbf{X}_{\mu_2}))$  holds, and thus  $\bar{P}(\mathbf{X}_{\mu_1}) \leq \bar{P}(\mathbf{X}_{\mu_2})$ , meaning that  $\bar{P}(\mathbf{X}_\mu)$  is a decreasing function of  $\mu > 0$ . Based on the above property, a bisection method can be used to find the  $\mu > 0$  satisfying  $\bar{P}(\mathbf{X}_\mu) = P_t$  as  $\mu^*$ .

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**Algorithm 1** The algorithm to solve (15).

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- 1: compute  $G_{klu}, \forall k, l, u$ .
  - 2:  $\mu_{\min} = 0; \mu_{\max} = 1$ ; compute  $\bar{P}(\mathbf{X}_{\mu_{\max}})$ ;
  - 3: **while**  $\bar{P}(\mathbf{X}_{\mu_{\max}}) \geq P_t$  **do**
  - 4:    $\mu_{\max} = 2\mu_{\max}$ ; compute  $\bar{P}(\mathbf{X}_{\mu_{\max}})$ ;
  - 5: **end while**
  - 6: **while**  $\mu_{\max} - \mu_{\min} > 0$  **do**
  - 7:    $\mu = \frac{\mu_{\max} + \mu_{\min}}{2}$ ; solve (22) for  $\mathbf{X}_\mu$ ;
  - 8:   **if**  $P_t - \epsilon \leq \bar{P}(\mathbf{X}_\mu) \leq P_t$  **then**
  - 9:     go to line 12;
  - 10:   **else if**  $\bar{P}(\mathbf{X}_\mu) > P_t$  **then**
  - 11:      $\mu_{\min} = \mu$ ;
  - 12:   **else**
  - 13:      $\mu_{\max} = \mu$ ;
  - 14:   **end if**
  - 15: **end while**
  - 16: compute the  $\mathbf{S}$  corresponding to  $\mathbf{X}_\mu$  and output it as an (at least approximately) optimum for (P1).
- 

The overall procedure to solve (16) is shown in Algorithm 1, where  $\epsilon > 0$  is a small prescribed tolerance. It can be shown in a similar way as in [15] that the finally produced  $\mathbf{X}_\mu$  is either equal to (if  $\bar{P}(\mathbf{X}_\mu) = P_t$  is satisfied), or a close approximation (if  $P_t - \epsilon \leq \bar{P}(\mathbf{X}_\mu) < P_t$  is satisfied) for  $\mathbf{X}^*$ . Moreover, the indicator variables in  $\mathbf{X}_\mu$  are either 0 or 1. Therefore, the  $\mathbf{S}$  corresponding to  $\mathbf{X}_\mu$  is either optimum or approximately optimum for (15). After finding this optimum  $\mathbf{S}$ , the optimum subcarrier assignment and source/relay power allocation can be computed accordingly. It can readily be shown that Algorithm 1 has a polynomial complexity with respect to  $K$  and  $U$ .

#### IV. NUMERICAL EXPERIMENTS

Consider the relay-aided downlink OFDMA system illustrated in Fig. 2.  $U = 5$  users are served and the users are randomly and uniformly distributed in a circular region of radius 50 m. To evaluate the maximum sum rate of the adopted protocol for each fixed set of system parameters, 500 random realizations of channels are generated. For each realization, the user coordinates are first randomly generated, and then the channels are generated in the same way as in [19].

When  $K = 32$  and  $P_t/\sigma^2$  increases from 15 to 25 dB, the average  $R_{\text{nov}}$  and  $R_{\text{ben}}$  which are the optimum sum rates for the two protocols, respectively, are shown in Fig. 3. It can be seen that the average  $R_{\text{nov}}$  is always greater than the average  $R_{\text{ben}}$ . This illustrates the benefit of using the novel protocol.

When  $P_t/\sigma^2 = 20$  dB and  $K$  increases from 4 to 64, the average  $\frac{R_{\text{nov}}}{R_{\text{ben}}}$  are shown in Fig. 4. It can be seen that the

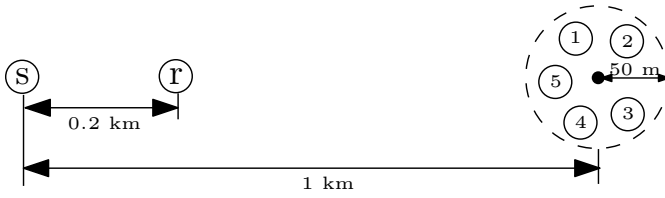


Fig. 2. The relay-aided downlink OFDMA system considered in numerical experiments.

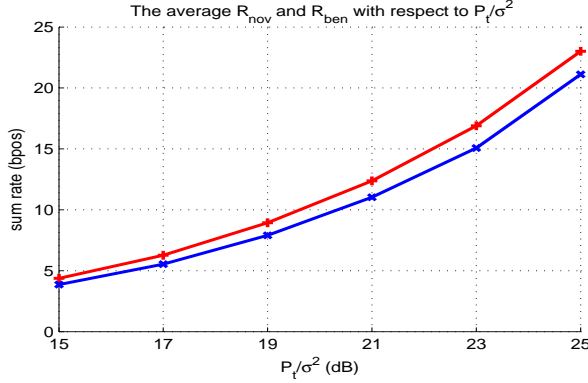


Fig. 3. The average  $R_{\text{nov}}$  and  $R_{\text{ben}}$  when  $K = 32$  and  $P_t/\sigma^2$  increases from 15 to 25 dB.

average  $\frac{R_{\text{nov}}}{R_{\text{ben}}}$  is always greater than 1, and the benefit of using the novel protocol increases as  $K$  increases. This indicates that the proposed protocol can better exploit the frequency-selective fading than the benchmark one for optimizing the subcarrier assignment to maximize the sum rate, especially when a big number of subcarriers is used.

## V. CONCLUSION

We have proposed a novel subcarrier-pair based opportunistic DF relaying protocol for downlink OFDMA transmission. Note that the proposed protocol truly improves the DF relaying itself. It is shown that the novel protocol leads to a maximum sum rate greater than or equal to that for the benchmark one. A polynomial-complexity RA algorithm has been developed for each protocol to maximize the sum rate of all users. Numerical experiments have illustrated that the novel protocol can lead to a much greater sum rate than the benchmark one.

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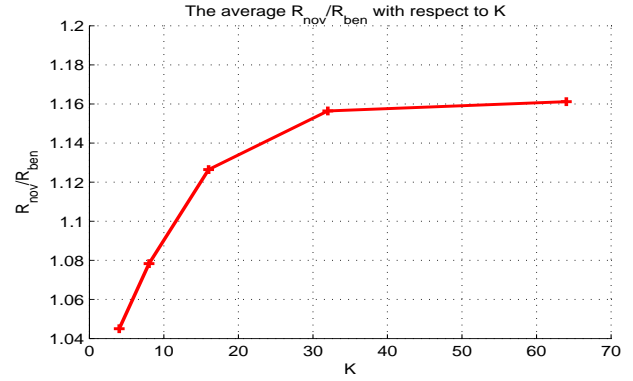


Fig. 4. The average  $\frac{R_{\text{nov}}}{R_{\text{ben}}}$  when  $P_t/\sigma^2 = 20$  dB and  $K$  increases from 4 to 64.

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